CONTINUUM MODELLING OF Al-Cu BIMETALLIC MATERIALS: STRESS-INTENSITY FACTOR CALCULATIONS

A thesis submitted to the

NATIONAL INSTITUTE OF TECHNOLOGY

In the partial fulfilment of the requirements for the degree of

Master of Technology

In

Mechanical Engineering (Steel Technology)

By

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May 2016

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National Institute of Technology Rourkela

May 2016

CERTIFICATE

This is to certify that the work presented in this thesis entitled "**Continuum Modelling of Al-Cu Bimetallic Materials: Stress Intensity Factor Calculations**" by "**Bhushan Jogi**", **Roll Number 214MM2498**, is a record of original research carried out by him under my supervision and guidance to the partial fulfilment of the requirements for the degree of "**Master of Technology**" in "**Mechanical Engineering (Steel Technology)**". Neither this thesis nor any part of it has been submitted for any diploma or degree to any institute or university in India or abroad.

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DECLARATION

I, **Bhushan Jogi**, **Roll Number 214MM2498** hereby declare that this thesis entitled "**Continuum Modelling of Al-Cu Bimetallic Materials: Stress Intensity Factor Calculations**" represents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, it contains no material previously published or written by another person, nor any material presented for the award of any other degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the thesis. I have also submitted my original research records to the scrutiny committee for evaluation of my thesis.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present thesis.

May 2016 **Bhushan Jogi**
NIT Rourkela

ACKNOWLEDGEMENT

It is my pleasure to take the opportunity of expressing my sincere gratitude to all those people who provided their support, collaboration, encouragement to carry out my thesis work. This project helped me a lot to extract out practical knowledge from theoretical work.

First of all I would like to thanks my supervisor, **Prof. Natraj Yedla,** Asst. Professor, Department of Metallurgical and Materials Engineering, NIT Rourkela, for his invaluable guidance and help in my thesis work. I also thank him for guiding me for every part of my work, for helping me to improve upon my mistakes all through the project work and for his kind cooperation, inspiration and providing experimental expertise require in my work.

I would like to extend my thanks to **Dr. Subhash Chandra Mishra,** Head of the Department, Department of Metallurgical and Materials Engineering, NIT Rourkela for providing the opportunity and facilities to pursue this work at the institute.

Bhushan Jogi

ABSTRACT

This project mainly focus with the study of mechanical properties of Al-Cu bimetallic material with the implementation of ANSYS Mechanical APDL simulation software. Al-Cu bimetals have wide applications in different fields such as in electrical, electronic & piping industries, heat engines, thermostat, thermometer, electrical devices, etc. It has been beneficial to characterise its mechanical properties which would be helpful to extend its applications for a variety of purposes. Simulation studies first has carried out for pure aluminium and pure Copper material individually. Determination of Stress Intensity Factor (SIF) in Mode-I loading by varying the kind of **cracks, crack length and applied stress**, which has been compared with already done researches on pure aluminium & pure copper material respectively, for the authentication of proposed method to obtain the Stress Intensity Factor (SIF) in mode I condition by the method of ANSYS Mechanical APDL codes. Next, Al-Cu bimetallic material has modelled in Mechanical APDL and the authenticated codes have been implemented to determine its Stress Intensity Factor (SIF) only in mode I loading conditions, under the variation of applied stress and crack length. Simulation is further succeeded by introduction of three different kinds of crack types: edge crack, central crack and circular crack with edge at the centre. Special case has taken by generating crack on either regions first in Al side and other on Cu side. In the last all the results have been concluded under Linear Elastic Fracture Mechanics (LEFM) by plotting graphs of **SIF Vs Applied stress and SIF Vs Crack length** for comparison with the theoretical values.

Key Words: Linear Elastic Fracture Mechanics (LEFM), Stress Intensity Factor (SIF), Finite Element Method, ANSYS_15.0 Mechanical APDL.

CONTENTS

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K	Stress Intensity Factor.
K_{I}	Stress Intensity Factor in mode I loading.
K_{II}	Stress Intensity Factor in mode II loading.
K_{III}	Stress Intensity Factor in mode III loading.
K_c	Fracture toughness.
c	hole radius.
ρ	radius of curvature at the tip of the hole.
E	Young's Modulus.
v	Poisson's ratio.
G	Strain energy release rate.
S, σ	Applied stress.
γ	Surface energy.
G_p	Plastic energy dissipation per unit area of crack growth.
a	Half a crack length.
W	Width of sheet.

CHAPTER 1

INTRODUCTION

1.1 Overview: In the field of materials characterisation, a lot of studies have been done on individual materials. Right from their extraction to their workable state in variety of applications we have characterised materials like Aluminium, copper, nickel, tungsten, etc. But when it comes to characterise the fractured forms of these materials, it becomes quite difficult as well as more interesting to invade their properties at that time. Materials specifically talking about pure metals, they tends to exhibit unexpected trends in their strengths and behaviours. Hence, we have chosen the most extensively used metals i.e. Pure Aluminium and Pure copper to economically simulate them by the method of Finite Element Method, (FEM) with the help of ANSYS Mechanical APDL tool. Once the metal got fractured its strength has been defined by the most important parameter i.e. Stress Intensity Factor (SIF).

Therefore, we have stressed in this thesis for the calculation of mode I (tensile loading condition) Stress Intensity Factor. Also, the target material for the due course of this work is Al-Cu bimetals.

1.2 Theory: Before moving directly to the simulation part. It has been quite handy to know about the theoretical aspects like phenomenon and fundamentals behind the calculations of Stress Intensity factor. So the peak of the fundamentals starts from the Linear Elastic Fracture Mechanics (LEFM). LEFM ultimately describes the equation and the variables on which the Stress Concentration Factor depends for all the three modes individually. Mode I, Mode II and Mode III are three possible cases in the calculations of SIF.

- ***Linear Elastic Fracture Mechanics:*** Unfortunately, the structural design on the basis of the basis of the tensile strength of the material resulted in many failures. Because the effect of stress-raising corners and holes on the strength of a particular structure was not appreciated by engineers. These failure result to the emergence of the field of ***"fracture mechanics".*** ***LEFM*** attempts to characterize a metal resistance to fracture.

- ***Finite Element Procedure:*** In our concerned software **ANSYS_15.0**, any problem is organised into three blocks: the pre-processor, the processor and the post-processor. In the pre-processor, the model is built defining the geometry, material properties and element type. Also, loads and boundary conditions are entered in the pre-processor, but the may be entered during the solution phase. With these details, the processor can compute. Next the algebraic equations formed by the model are solved and the solutions are obtained. In the last block, the post processor derived the results.

- ***Manufacturing of Bimetals:*** Bimetals are basically the combination of two different metals in the layered form. It can be fabricated through various kinds of methods: cast surfacing, continuous casting, centrifugal casting, multi-layer surfacing, surfacing under a layer of hot slag, electro-slag surfacing using liquid metal, Broad-layer electro-slag surfacing, vertical electro-slag surfacing, explosive cladding(welding),stacked rolling, cold surfacing, joining of interface through welding, brazing, soldering. Here we have considered our material Al-Cu bimetal manufactured by joining of interface with very thin layer of welding which has been significantly small can be neglected.

1.3 Applications: Al-Cu bimetals have wide applications in different fields such as in electrical, electronic & piping industries, heat engines, electrical & electronic devices. Following are the examples among them:

- In electrical engineering and electronics for the production of wires and electronic components.
- In electric circuits for unbreakable contacts

- In machine parts and system components due to better performance in stamping, bending and welding.

1.4 Objectives :

- To design ANSYS codes for obtaining the **(SIF)** stress intensity factor value in **mode I** for any material.
- To obtain **(SIF)** stress intensity factor values in **mode I** for pure aluminium and pure copper respectively with variation in the **applied stress and crack** length for each of the three kinds of crack.
- To authenticate the trial ANSYS code with the previous researches done on pure aluminium and pure copper materials.
- To obtain the **(SIF)** stress intensity factor values in **mode I** for Al-Cu bimetallic material with the variation in applied stress, crack length for each of the three kinds of crack.
- To obtain the **(SIF)** stress intensity factor values in **mode I** for Al-Cu bimetallic material for a special case.

CHAPTER 2

LITERATURE SURVEY

2.1 Overview:

2.1.1 Finite Element Method (FEM): The finite element method (FEM) has developed along two paths. From mathematical point of view, it is the method of constructing a function that makes the potential energy a minimum. From the engineering point of view, it is a method of assembling structural elements, which can be separately analysed, into a global equation of equilibrium for the structure [1].

2.1.2 Bimetal Material: Bimetals are one of the simplest sorts of metal composites and as it is clear from their name, are combined from two metals or metal alloys. These two metals or metal alloys form two layers which a metallurgical bonding (metal bonding) between them is established constitute a single piece composite, purpose of bimetals production is to create the integrated components comprises of two metals so that each metal offer its unique properties. [2]

2.1.3 Linear Elastic Fracture Mechanics: It attempts to characterise a material's resistance to fracture- "its toughness". In 1913, Inglis showed that the local stresses around a corner or hole in a stressed plate could be very large than the average applied stress. Presence of sharp corners, cracks or notches was responsible to concentrate applied stress to these points. Using elasticity theory, Inglis showed that the degree of stress magnification at the edge of the hole is depended on radius of curvature of the hole.

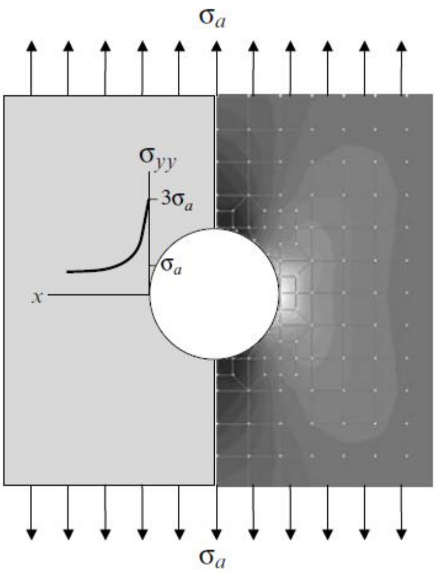


Fig 2.1 Schematic of plate with a hole.

Mathematically,

$$\boldsymbol{K = 1+2\sqrt{\frac{c}{\rho}}} \qquad \textbf{(2.1)}$$

It should be noted that the stress concentration factor did not depend on the absolute size or length of the hole but only on the ratio of size to radius of curvature.

2.1.3.1 Griffith's Criterion: Fracture Mechanics was invented around World War I by A.A Griffith to describe the failure of brittle materials [8]. His work was motivated on two contradictory facts:

- The stress needed to fracture bulk glass was around 100 MPa.
- The theoretical stress needed for breaking atomic bonds was approximately 10,000 MPa

A theory was needed to sort out these conflicting observations. Also, experiments on glass fibre that he himself conducted suggested that the fracture stress increases as the fibre diameter decreases. Hence Griffith showed that the product of the square root of the flaw length and the stress at the fracture was nearly constant, which is expressed by the equation:

$$\sigma_f\sqrt{a} \approx C \tag{2.2}$$

An explanation of this relation in terms of linear elastic theory was problematic. Linear elasticity theory says that stress (hence the strain) at the crack tip of a sharp flaw in a material is infinite. To avoid it, he developed a thermodynamic approach to explain it.

The growth of crack requires the creation of two new surfaces and hence an increase in the surface energy. Briefly the approach was:

- Calculate the potential energy stored in perfect specimen under uniaxial tensile load.
- Fix the boundary so that applied load does no work and then induce a crack into the specimen. The crack relaxes the stress and hence reduces the elastic energy near the crack faces. Other side crack increases the total surface energy of the specimen.
- Calculate the change in free energy (Surface energy – Elastic energy) as a function of crack length. Failure occurs when the free energy attains a peak at the critical length, beyond which the free energy decreases by increasing the crack length i.e. by causing fracture. Griffith concluded that:

$$C = \sqrt{\frac{2E\gamma}{\pi}} \tag{2.3}$$

2.1.3.2 Irwin's Modification: Two reasons are:

- In the actual structural materials the level of energy needed to cause fracture was orders of magnitude higher than the corresponding surface energy.
- In structural materials there are always some inelastic deformations around the crack front that would make the assumption of linear elastic medium with infinite stresses at the crack tip which is highly unrealistic. [9]

Hence a dissipative term has to be added to the energy balance relation devised by Griffith for brittle materials. In physical terms additional energy is needed for crack growth in ductile materials when compared to brittle materials.

Irwin's strategy was to partition the energy into two parts:

- The stored elastic strain energy which is released as a crack grows. This is thermodynamic driving force for fracture.
- The dissipated energy which includes plastic dissipation and the surface energy .The dissipation energy provides the thermodynamic resistance to fracture. Then the total energy.

$$\boldsymbol{G = 2\gamma + G_p} \tag{2.4}$$

Eventually a modification of Griffith's solid theory emerged from this work: a term stress intensity replaced strain energy release rate and a term fracture toughness replaced surface weakness energy. Both of these terms are simply related to the energy terms:

$$\boldsymbol{K_I = \sigma\sqrt{\pi a}} \tag{2.5}$$

And

$$\boldsymbol{K_c = \sqrt{EG_c}} \quad \textbf{(for plane stress)} \tag{2.6}$$

$$\boldsymbol{K_c = \sqrt{\frac{EG_c}{1-\nu^2}}} \quad \textbf{(for plane strain)} \tag{2.7}$$

It is important to identify the fact that fracture parameter K_c when measured under plane stress and plane strain. We must note the expression for K_I in equation 2.4 will be different for geometries other than the centre cracked infinite plate. Consequently it is necessary to introduce a dimensionless correction factor, Y, in order to characterise the geometry:

$$\boldsymbol{K_I = Y\,\sigma\sqrt{\pi a}} \tag{2.8}$$

Where Y is the function of crack length and width of sheet given by:

$$Y\left(\frac{a}{W}\right) = \sqrt{\sec\left(\frac{\pi a}{W}\right)} \qquad \textbf{(2.9)}$$

Engineers became habitual to using K_{Ic} to characterise the fracture toughness, a relation has been used to reduce J_{Ic} to it:

$$K_{Ic} = \sqrt{E^{*} J_{Ic}} \qquad \textbf{(2.10)}$$

Where E^{*} = E for plane stress and E^{*} = $\frac{E}{1-\upsilon^2}$ for plane strain

2.1.4 Stress Intensity Factor, *K*:

SIF is used in fracture mechanics to guess the stress state i.e. stress intensity near the tip caused by the residual load [3]. This concept is usually used for homogeneous, linear elastic material and for establishing a failure criterion of brittle materials, & also a technique for critical damage tolerance. It can be applied to material that exhibit small scale yielding at a crack tip.

The magnitude of K depends on:

- **Sample geometry**
- **Size and location of the crack**
- **Magnitude of load**
- **Distribution of load**

The stress Intensity factor is a single-parameter characterization of the crack tip stress field.

Linear Elastic theory predicts that the stress distribution (σ_{ij}) near the crack tip, in polar coordinates (r,θ) with origin at the crack tip has the form [4]

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \textit{Higher order terms} \qquad \textbf{(2.11)}$$

Where, K is the stress intensity factor, f_{ij} is the dimensionless quantity which varies with load and geometry.

2.1.4.1 Stress Intensity Factor for various modes: There are three linearly independent cracking modes in fracture mechanics. These are categorized as Mode I, II or III. Mode I is an opening (tensile) mode where the crack surface moves apart. Mode II is a sliding (in plane shear) mode where the crack surfaces slide over one another. Mode III is a tearing (anti-plane shear) mode where the crack surface moves relative to one another. Mode I is the most common load type encountered in engineering design. These factors are formally defined as: [5]

$$K_{\mathrm{I}} = \lim_{r\to 0} \sqrt{2\pi r}\, \sigma_{yy}(r,\theta) \qquad (2.12)$$

$$K_{\mathrm{II}} = \lim_{r\to 0} \sqrt{2\pi r}\, \sigma_{yx}(r,\theta) \qquad (2.13)$$

$$K_{\mathrm{III}} = \lim_{r\to 0} \sqrt{2\pi r}\, \sigma_{yz}(r,\theta) \qquad (2.14)$$

2.1.4.2 Relationship to energy release rate: The strain energy release rate (*G*) for crack under mode I loading is implied as:

$$G = K_I^2 \left(\frac{1-v^2}{E}\right) \qquad (2.15)$$

The material is assumed to be an isotropic, homogeneous and linear elastic. Plain strain has been assumed and the crack has been assumed to extend along the direction of the initial crack. For plain stress condition:

$$G = K_I^2 \left(\frac{1}{E}\right) \qquad (2.16)$$

2.1.5 Determination of K by Finite Element Analysis: For a complicated geometry or loading, the exact solution to the linear elasticity problem cannot be determined by direct means and we must turn to numerical methods. There are several methods that can be used to determine the stress intensity factor.

The oldest method is the direct calculation of the strain energy release rate. A stress analysis can be performed for various lengths *L* of a crack but the same external load. For each analysis, the stored energy is readily calculated. A curve of U versus *L* is plotted and the slope of this curve is the strain energy release rate. The stress intensity factor *K* is then determined by the fundamental definition. The accuracy of this method is limited since differentiation in order to determine the slope magnifies the error in the finite element calculation of displacements [6].

Crack Opening Displacement Method: Accuracy can be improved by using the finite element method to determine the crack opening displacement. This requires a detailed knowledge of displacements near the crack tip from the continuum mechanics analysis in advance of the finite element analysis.

$$\boldsymbol{K = \sqrt{\frac{E}{t}\frac{\partial U}{\partial L}}} \tag{2.17}$$

For example in the case of the centre crack in a thin sheet displacement on the crack surface is given by:

$$\boldsymbol{u_y = \frac{2K}{E\sqrt{\pi a}}\sqrt{a^2 - x^2}} \tag{2.18}$$

If r is the distance from the crack tip,

$$\boldsymbol{u_y = \frac{2K\sqrt{2r}}{E\sqrt{\pi}} * \sqrt{1 - \frac{r}{2a}} \approx \frac{4K\sqrt{r}}{E\sqrt{2\pi}}} \tag{2.19}$$

The displacement calculated at a node on the crack face near the crack tip can be used to determine u_y and the location of the node determines *r*. Equation 2.19 is then used to determine *K*. The accuracy of this calculation is strongly affected by the accuracy of the finite element model near the crack tip.
An alternative is the following procedure, with $\theta = \pi$, or neglecting *r/a*, we have $(u_y \equiv v)$ for plane stress [7]

$$\boldsymbol{K = \frac{E\sqrt{2\pi}}{4}\frac{v}{\sqrt{r}}.} \tag{2.20}$$

2.1.6 Description of Materials: Pure Aluminium, pure copper and Al-Cu bimetals are the three materials which has discussed in this thesis.

- ***Aluminium:*** Aluminium is a white-silvery ductile metal. It is the most abundant metal on the earth's crust, and is extracted from bauxite majorly. Important properties are superior malleability, excellent corrosion resistant, high strength, good electrical conductivity and easy machining.

- ***Copper:*** Copper is reddish-brown metal. It is soft, shiny and very ductile in nature.it is mainly extracted from copper sulphides. Important properties are high ductility, high thermal and electrical conductivity.

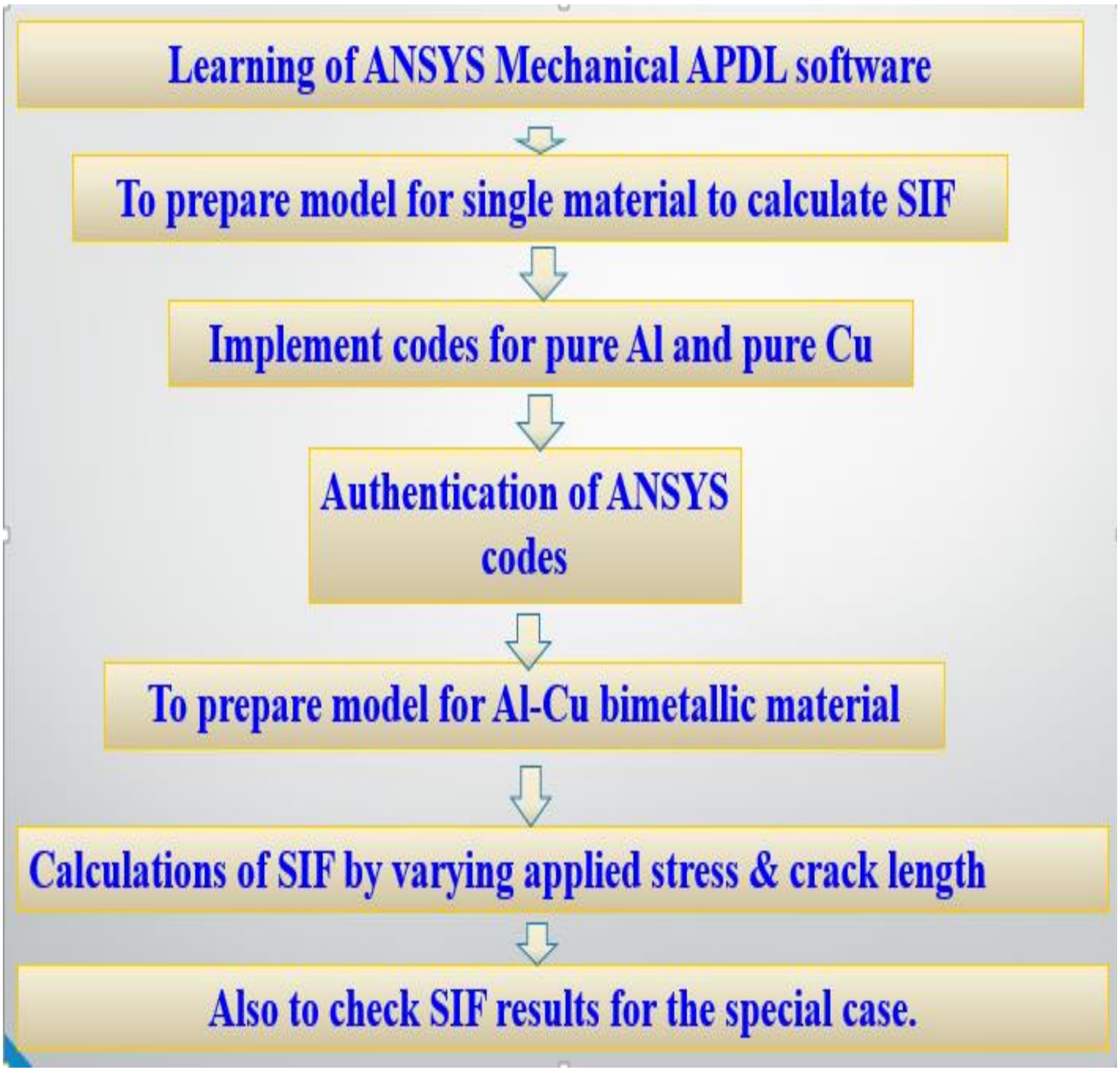
- ***Al-Cu Bimetals:*** It is the combination of pure aluminium and pure copper in the layered form. It can be obtained from following way:

  1. Continuous casting
  2. Centrifugal casting
  3. Stacked rolling
  4. Cold surfacing
  5. Multi-layered surfacing
  6. Explosive welding
  7. Electro-slag surfacing in liquid metal
  8. Broad layer electro-slag surfacing
  9. Vertical electro-slag surfacing.

CHAPTER 3

SIMULATION PROCEDURE

- **Work plan flow chart**



3.1 ANSYS codes for single material model: Determination of Stress Intensity Factor (SIF) for pure aluminium material as well as pure copper material.

Table 3.1 Material properties of pure Al & pure Cu.

Property	Pure aluminium	Pure copper
Density (g/cm^3)	2.76	8.96
Modulus of Elasticity (GPa)	69	119
Poisson ratio	0.35	0.34
UTS (MPa)	310	220
Yield Stress (MPa)	7-11	69-120

a. **ANSYS Codes: SIF by Crack Opening Displacement.**

- ***Set job name and preferences.***

  C FILE>CHANGE JOBNAME

  T a jobname

  SELECT NEW LOG Yes

  OK

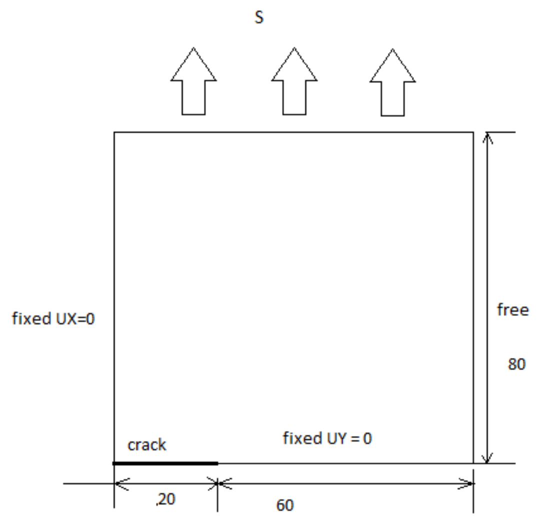


Fig 3.1 Line Diagram of the cracked sheet.

- ***Establish element type and material properties.***

A cracked sheet is loaded in tension. Because of double symmetry, we can use one quarter for the analysis with symmetric conditions on the edge x=0 and y=0.

PREPROCESSOR>ELEMENT TYPE>ADD

C ADD

C SOLID

C QUAD 8 node 183

C OK

C OPTIONS

Plane stress should be selected

C OK

C CLOSE

C PREPROCESSOR>MATERIAL PROP>MATERIAL MODELS

C STRUCTURAL

C LINEAR

C ELASTIC

C ISORTOPIC

T ….for EX parameter

C in PRXY box

T…..for PRXY parameter

C OK

C MATERIAL>EXIT

- ***Establish geometry and mesh the object***

  The upper right quarter of the cracked sheet. The origin as a KEYPOINT is placed at the crack tip by using two rectangles and then combining them into one material body. The origin must be at the crack tip.

  C PREPROCESSOR>MODELING>CREATE>AREAS

  C RECTANGLE>BY DIMENSION

  T -20 0 0 60 for X1, X2, Y1, Y2

  C APPLY

  T 0 60 0 80 for X1, X2, Y1, Y2

  C OK

  PLOT CTRLS>NUMBERING

C box after Key point Numbers to turn them ON

C box after Line Numbers to turn them ON

C box after Area Numbers to turn them ON

C box after Nodes Numbers to turn them ON

C OK

C PREPROCESSOR>MODELING>OPERATE>BOOLEANS

>ADD>AREAS

C PICK ALL

C PREPROCESSOR>MESHING>SIZE CONTROLS

C CONCENTRATE KPs> CREATE

PICK crack tip at the origin

C OK

C PREPREOCESSOR>MESHING >MESH>AREA>FREE

C PICK ALL [close warning message]

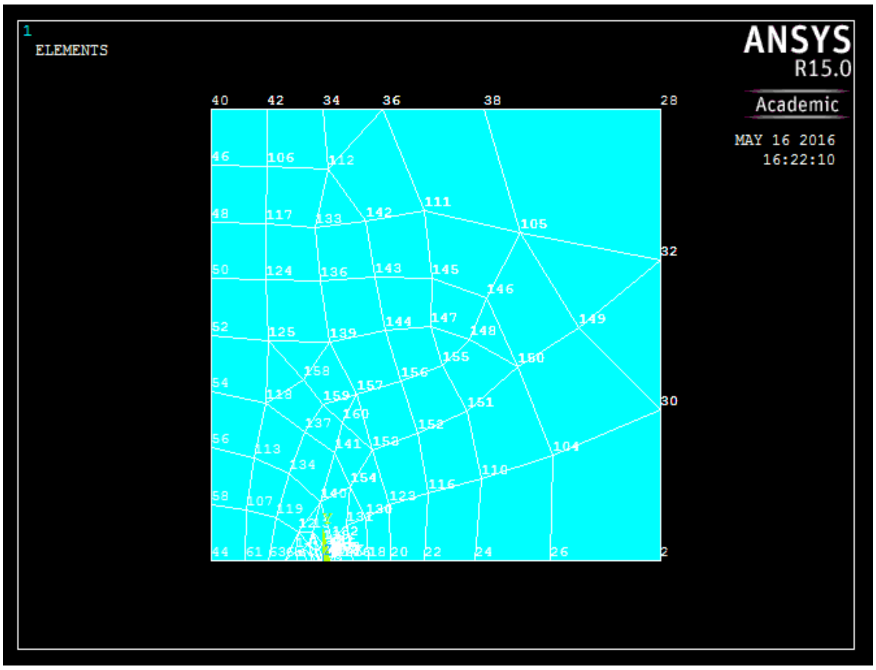


Fig 3.2 Meshing snapshot of single material model.

- ***Apply boundary conditions and solve foe displacements and stresses***

PLOT>LINES

SOLUTION>DEFINE

LOADS>APPLY>STRUCTURAL>DISPLACEMENT>

ON LINES

C bottom right edge of the model (L9)

C APPLY

C UY

Enter 0 for the value of the displacement components

C APPLY

C left edge of the model (L4)

C OK

C UX

Enter 0 for the value of the displacement components (Symmetry Conditions)

C OK

C SOLUTION>DEFINE>LOADS>APPLY>STRUCTURAL> PRESSURE>ON LINES

C top edge of both of the original two elements (L3and L10)

C OK

Enter -….for Load stress value [negative for tension]

C OK

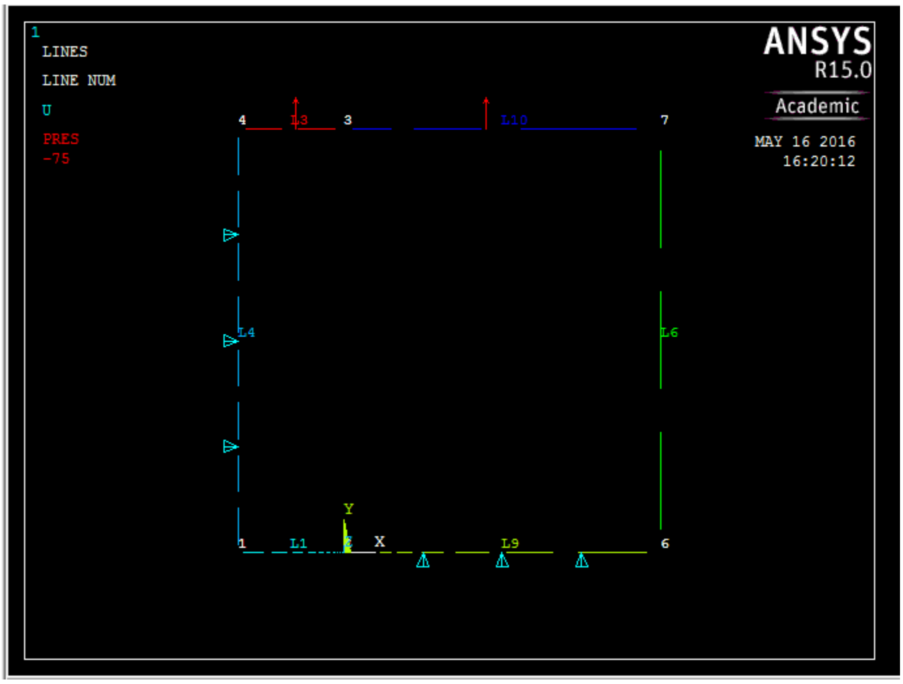


Fig 3.3 Boundary conditions snapshot of single material model.

C SOLUTION>SOLV CURRENT LS

C CLOSE on information window

C OK in SOLVE window

C YES in warning window

CLOSE on information that solution is complete

- ***Display results and calculate stress intensity factor***

  C GENERAL POST PROC>PLOT RESULTS> DEFORMED SHAPE

  C OK

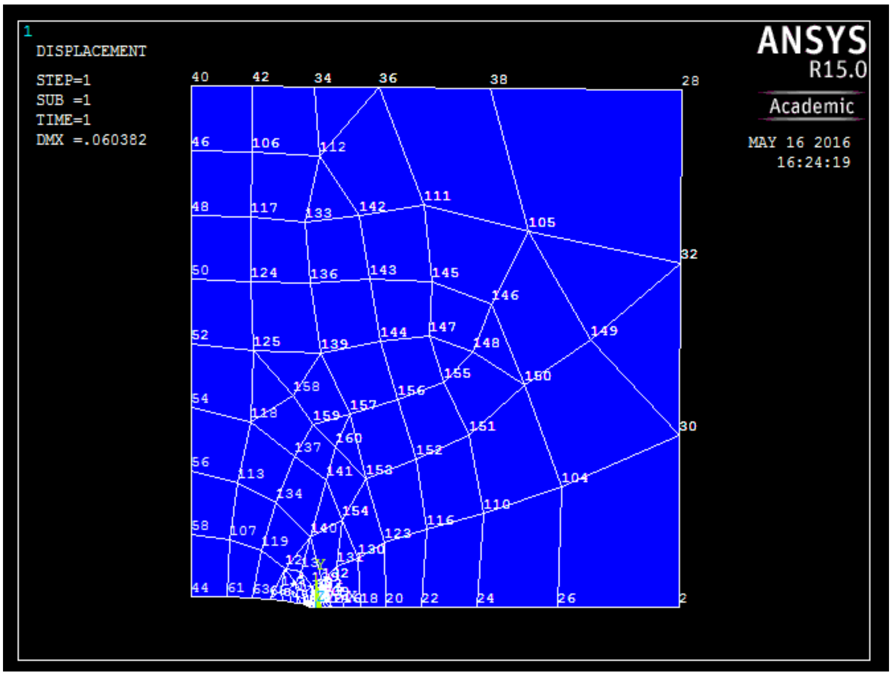


Fig 3.4 Deformed shape with meshing snapshot of single material model.

Select PLANE STRESS from the menu for KPLAN

Select HALF-SYMM B.C. from the menu for KCSYM

C OK produces listing showing K_I.

CLOSE

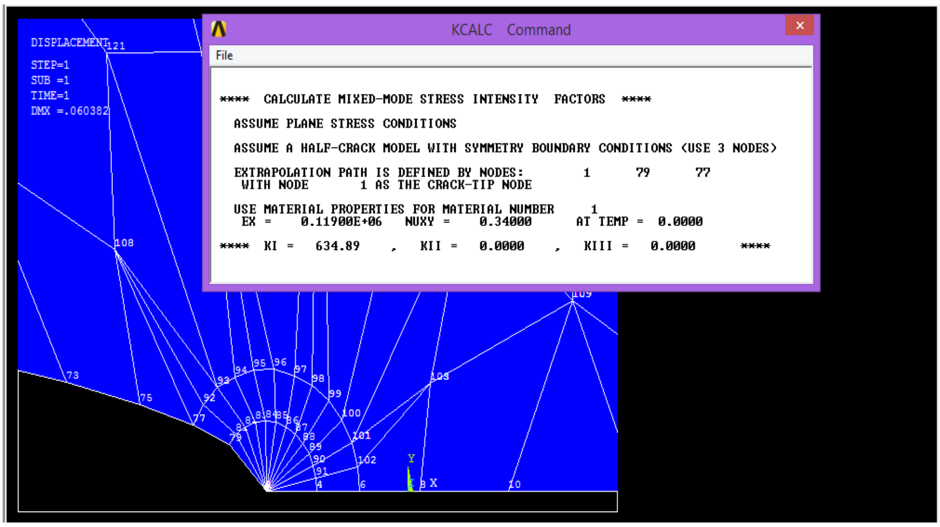


Fig 3.5 Snapshot representing value of mode I Stress Intensity Factor for single material

3.2 ANSYS codes for Al-Cu Bimetallic material model:

Determination of Stress Intensity Factor (SIF) for Al-Cu Bimetallic material.

a. **ANSYS Codes: SIF by Crack Opening Displacement.**

- ***Set job name and preferences.***

  C FILE>CHANGE JOBNAME

  T a jobname

  SELECT NEW LOG Yes

  OK

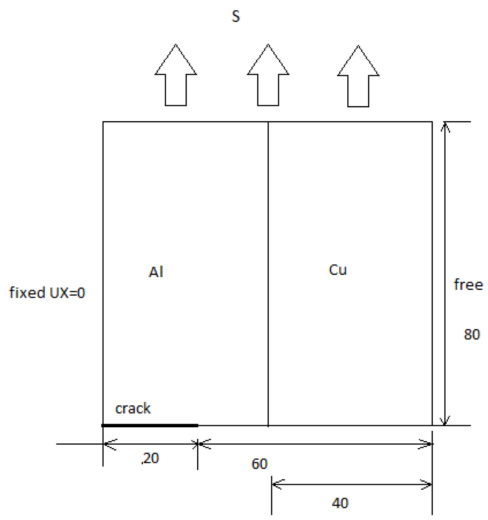


Fig 3.6 Line Diagram for Al-Cu Bimetallic Material model.

- ***Establish element type and material properties.***

A cracked sheet is loaded in tension. Because of double symmetry, we can use one quarter for the analysis with symmetric conditions on the edge x=0 and y=0.

PREPROCESSOR>ELEMENT TYPE>ADD

C ADD

C SOLID

C QUAD 8 node 183

C OK

C OPTIONS

Plane stress should be selected

C OK

C CLOSE

C PREPROCESSOR>MATERIAL PROP>MATERIAL MODELS

C STRUCTURAL

C LINEAR

C ELASTIC

C ISORTOPIC

T ….for EX parameter

C in PRXY box

T…..for PRXY parameter

C OK

C Material >New Model >Define Material ID>

T 2

C OK

C STRUCTURAL

C ELASTIC

C ISOTROPIC

T ….for EX

C in PRXY box

T ….for PRXY

C OK

C MATERIAL>EXIT

- ***Establish geometry and mesh the object***

The upper right quarter of the cracked sheet. The origin as a KEYPOINT is placed at the crack tip by using two rectangles and then combining them into one material body. The origin must be at the crack tip.

C PREPROCESSOR>MODELING>CREATE>AREAS

C RECTANGLE>BY DIMENSION

T -20 0 0 60 for X1, X2, Y1, Y2

C APPLY

T 0 20 0 80 for X1, X2, Y1, Y2

C APPLY

T 20 60 0 80 for X1, X2, Y1, Y2

C OK

PLOT CTRLS>NUMBERING

C box after Keypoint Numbers to turn them ON

C box after Line Numbers to turn them ON

C box after Area Numbers to turn them ON

C box after Nodes Numbers to turn them ON

C OK

C PREPROCESSOR>MODELING>OPERATE>BOOLEANS
>ADD>AREAS

C PICK ALL

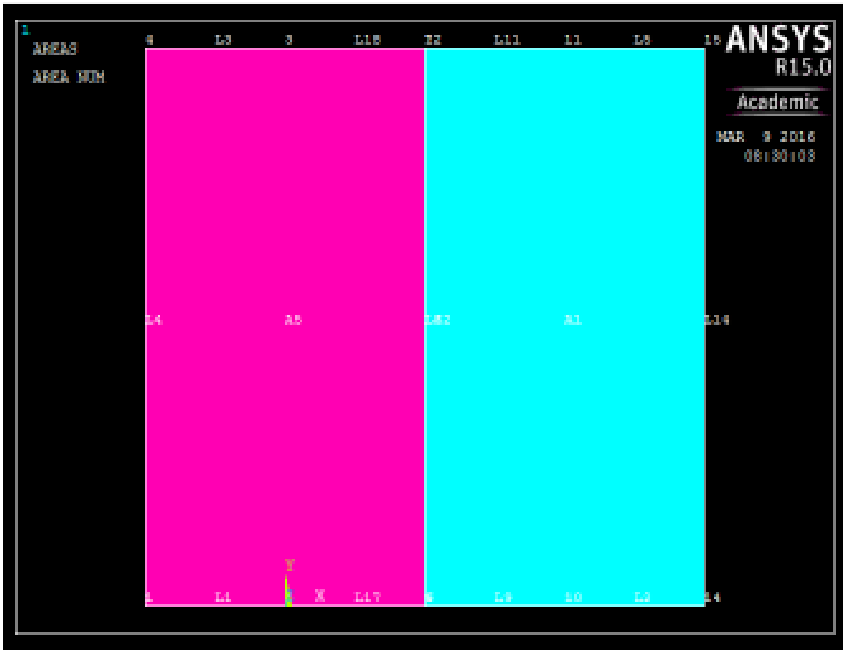


Fig 3.7 Snapshot of Al-Cu Bimetallic material model.

C PREPROCESSOR>MESHING>SIZE CONTROLS

C CONCENTRATE KPs> CREATE

PICK crack tip at the origin

C OK

C PREPREOCESSOR>MESHING >MESH>AREA>FREE

C PICK ALL [close warning message]

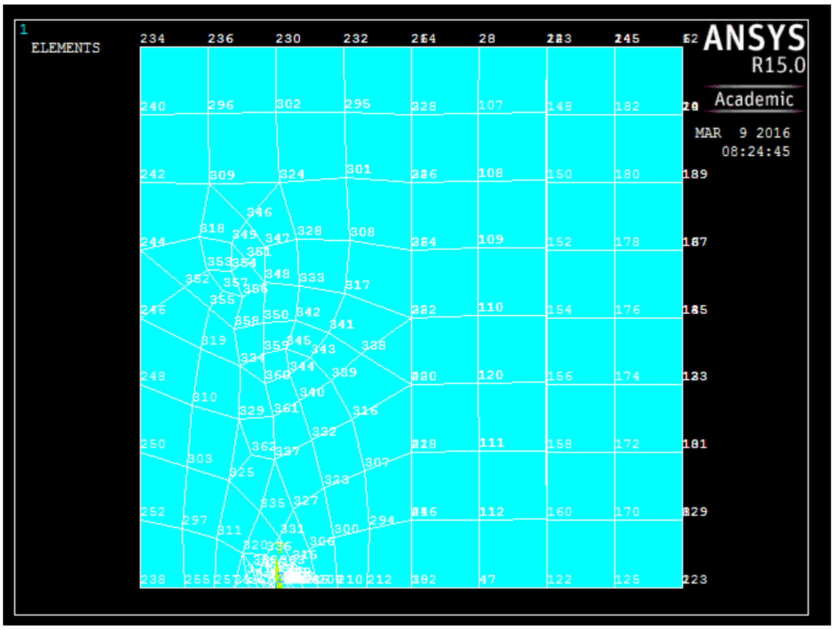


Fig 3.8 Meshing snapshot of Al-Cu bimetallic material model.

- ***Apply boundary conditions and solve for displacements and stresses***

  PLOT>LINES

  SOLUTION>DEFINE

  LOADS>APPLY>STRUCTURAL>DISPLACEMENT>

  ON LINES

  C bottom right edge of the model (L7, L9, L2)

  C APPLY

  C UY

  Enter 0 for the value of the displacement components

  C APPLY

  C left edge of the model (L4)

  C OK

  C UX

  Enter 0 for the value of the displacement components (Symmetry Conditions)

  C OK

  C SOLUTION>DEFINE>LOADS>APPLY>STRUCTURAL>

  PRESSURE>ON LINES

  C top edge of both of the original two elements (L3, L8, L11, L5)

  C OK

  Enter -….for Load stress value [negative for tension]

  C OK

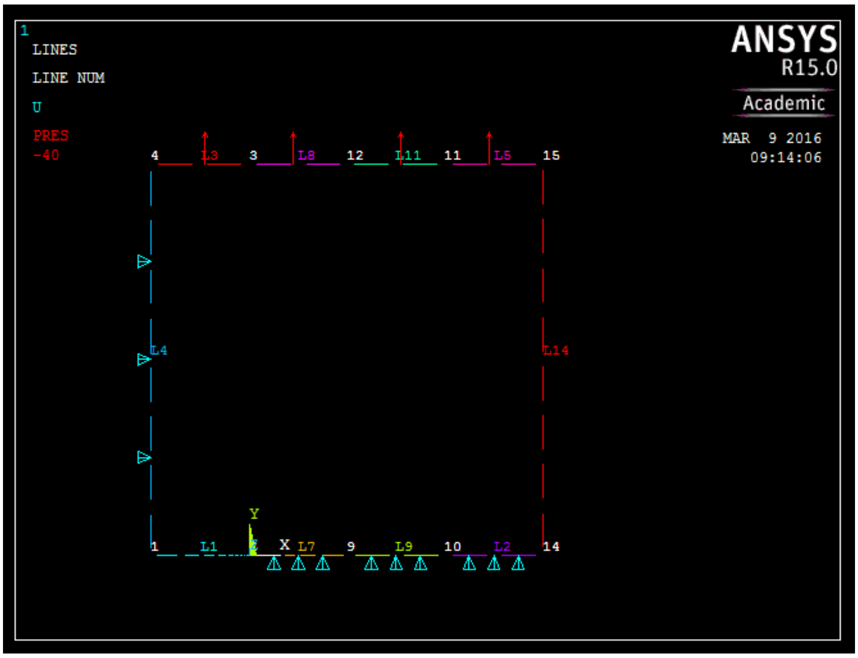


Fig 3.9 Boundary conditions snapshot of Al-Cu bimetallic material model

C SOLUTION>SOLV CURRENT LS

C CLOSE on information window

C OK in SOLVE window

C YES in warning window

CLOSE on information that solution is complete

- ***Display results and calculate stress intensity factor***

  C GENERAL POST PROC>PLOT RESULTS> DEFORMED SHAPE

  C OK

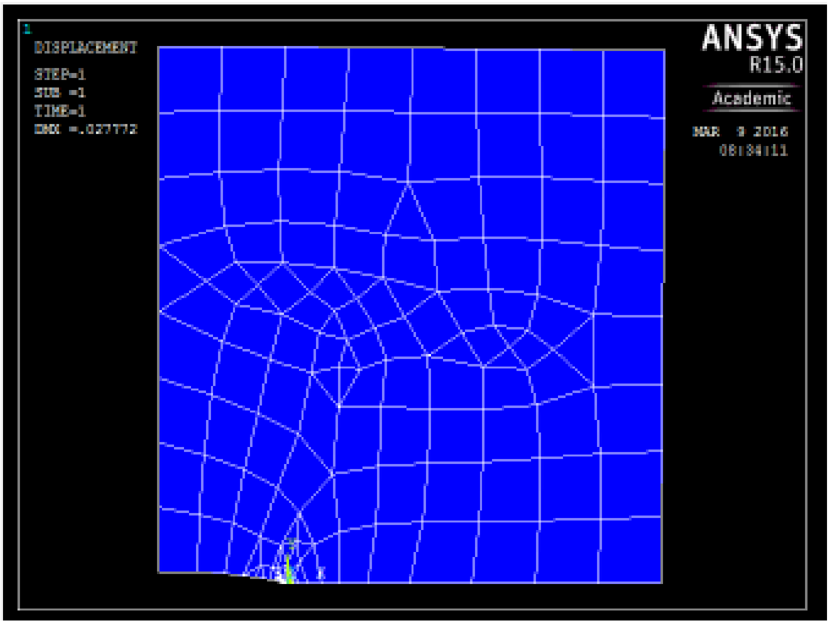


Fig 3.10 Deformed shape with meshing snapshot of Al-Cu bimetallic material model.

Select PLANE STRESS from the menu for KPLAN

Select HALF-SYMM B.C. from the menu for KCSYM

C OK produces listing showing K_I.

CLOSE

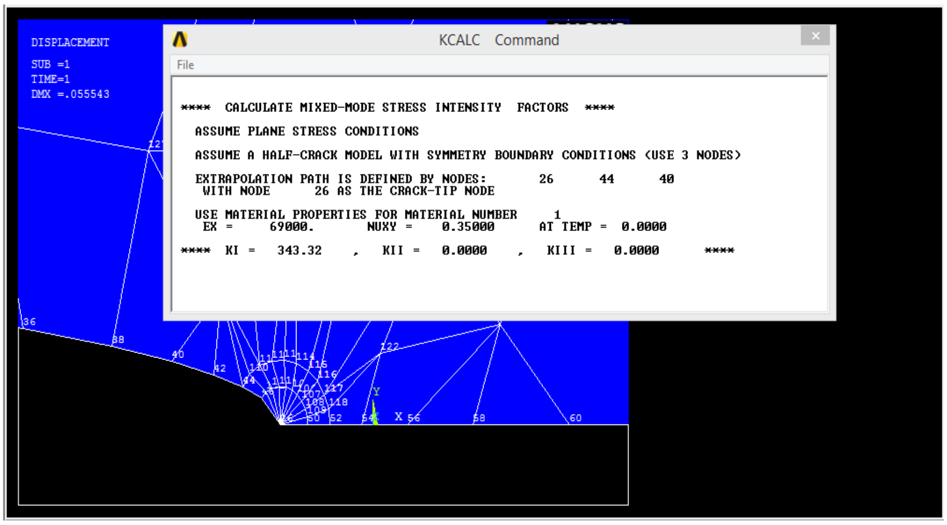


Fig 3.11 Snapshot representing value of mode I Stress Intensity Factor for Al-Cu Bimetallic material model.

- The applied stress values range have been decided on the basis of yield stress value of each of the materials: pure Al and pure Cu.

CHAPTER 4

RESULTS & DISCUSSIONS

4.1 Aluminium:

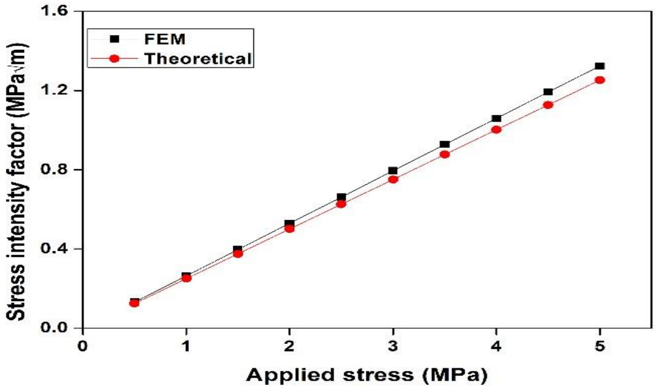
First of all we have done simulation on pure aluminium material for determining its Stress Intensity Factor (SIF) mode I using simulation tool ANSYS Mechanical APDL. Under the following different variations they are:

a. By varying crack type.
    - Edge crack.
    - Central crack.
    - Circular crack.
b. By varying applied stress.
c. By varying crack length.

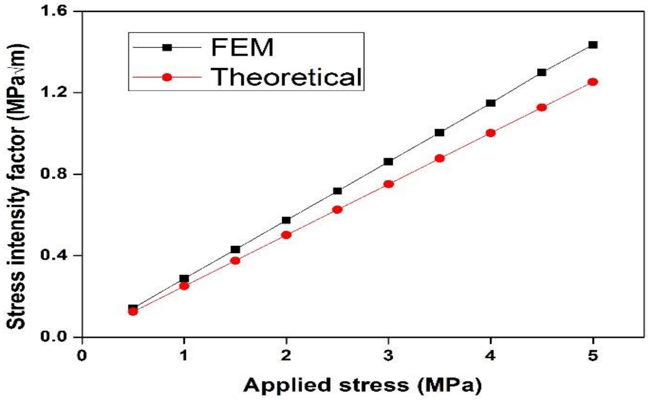
Table 4.1 Values of SIF for varying loading in pure aluminium material.

S. No	**Applied stress (MPa)**	**$(K_I)_{Theoretical}$ (in MPa.m$^{1/2}$)**	**Edge crack, $K_{I(FEM)}$ (in MPa.m$^{1/2}$)**	**Central crack, $K_{I(FEM)}$ (in MPa.m$^{1/2}$)**	**Circular crack with an edge, $K_{I(FEM)}$ (in MPa.m$^{1/2}$)**
1.	0.5	0.125	0.134	0.143	0.154
2.	1	0.250	0.264	0.287	0.309
3.	1.5	0.375	0.397	0.430	0.463
4.	2	0.501	0.530	0.574	0.618
5.	2.5	0.626	0.662	0.717	0.772
6.	3	0.75	0.795	0.8612	0.927
7.	3.5	0.877	0.927	1.004	1.081
8.	4	1.002	1.059	1.148	1.236
9.	4.5	1.127	1.192	1.291	1.390
10.	5	1.253	1.324	1.435	1.545

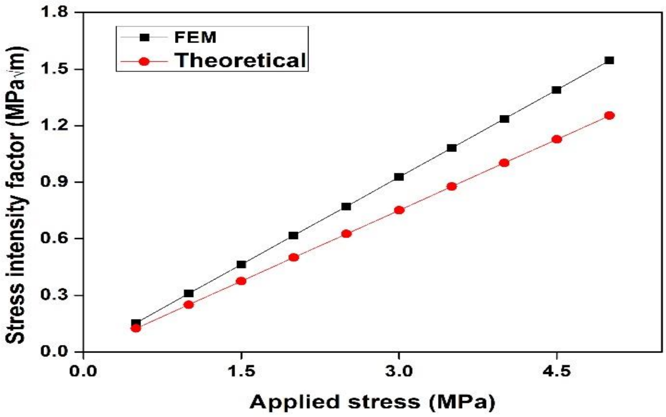
According to Table 4.1, the values of SIF through FEM (Finite Element Method) have found to be increasing with the increase in applied stress. Hence it has proved that K_I *i.e.* Stress Intensity factor around the tip of the crack surface is directly proportional to the applied stress. Again, according to table 4.1, the values of SIF through FEM have found to be changing with complexity of the crack. Hence it has proved that the geometry of crack influences the value of K_I. Stress Intensity factor has higher values for more complicated geometry because the residual stresses around the crack tip got increased.



(a)



(b)



(c)

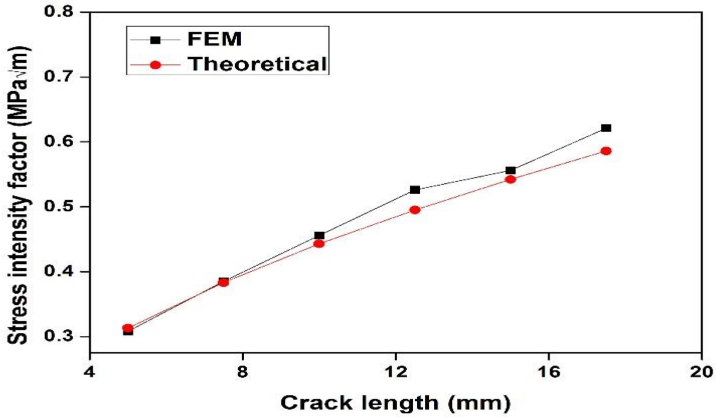
Fig 4.1 Stress Intensity Factor Vs Applied stress graph for pure aluminium material (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to Fig 4.1, the curve for theoretical values of K_I in each of the cases have found to be lied above the curve for FEM values of K_I .It is due to omission of geometrical factor in the theoretical calculations of Stress Intensity factor K_I . Again from the fig 4.1 it could be concluded that as the applied stress kept on increasing the difference between theoretical values and FEM values has gone increased.

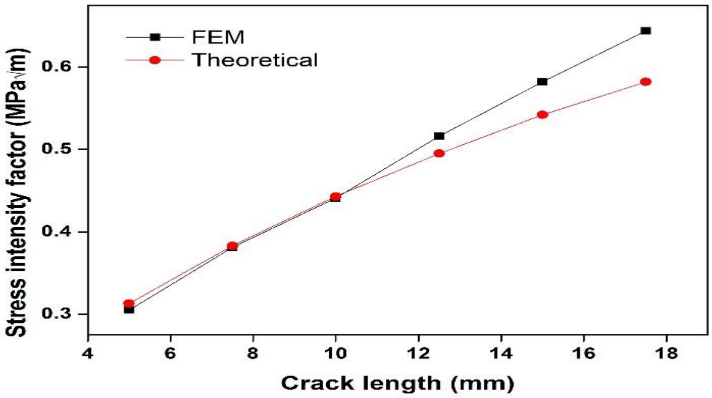
Table 4.2 Values of SIF for varying crack length in pure aluminium material.

S. No	**Crack length (in mm)**	**$K_{I(Theoretical)}$ (in MPa.m$^{1/2}$)**	**Edge crack $K_{I(FEM)}$ (in MPa.m$^{1/2}$)**	**Central crack $K_{I(FEM)}$ (in MPa.m$^{1/2}$)**	**Circular crack with an edge $K_{I(FEM)}$ (in MPa.m$^{1/2}$)**
1.	5	0.313	0.308	0.305	0.330
2.	7.5	0.383	0.385	0.381	0.407
3	10	0.443	0.456	0.441	0.477
4.	12.5	0.495	0.526	0.516	0.546
5.	15	0.542	0.556	0.582	0.623
6.	17.5	0.586	0.621	0.644	0.692

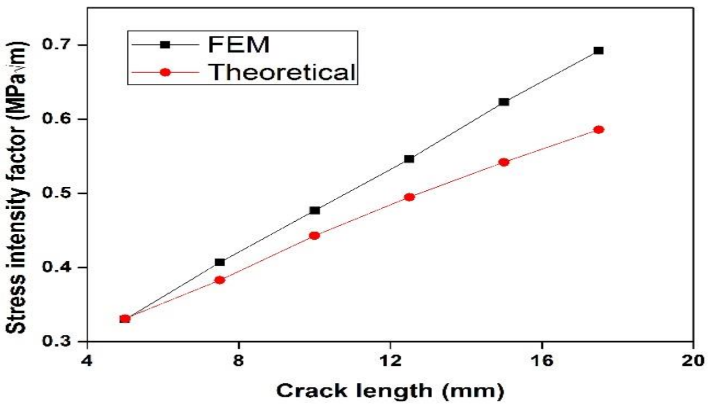
According to Table 4.2, the values of K_I through FEM have found to be increasing gradually with the increase in crack length. Hence it can be concluded that the value of K_I is directly proportional to the crack length. Therefore, magnitude of Stress intensity factor depends on the crack length. Again from table 4.2, as the crack geometry is changed with the each case values of K_I have found to be changed. It could be meant that the location of crack also plays the role in the Stress Intensity Factor.



(a)



(b)



(c)

Fig 4.2 Stress Intensity Factor Vs Crack length graph for pure aluminium material (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to Fig 4.2, again the curve for theoretical values has found to be lied above the curve for FEM values. Also, variation among the respective values on both the curve is uneven in the case (a) abruptly changing, (b) curves got intersected (c) moving apart. From the above interpretation, we could not predict the exact nature efficiently.

4.2 Copper

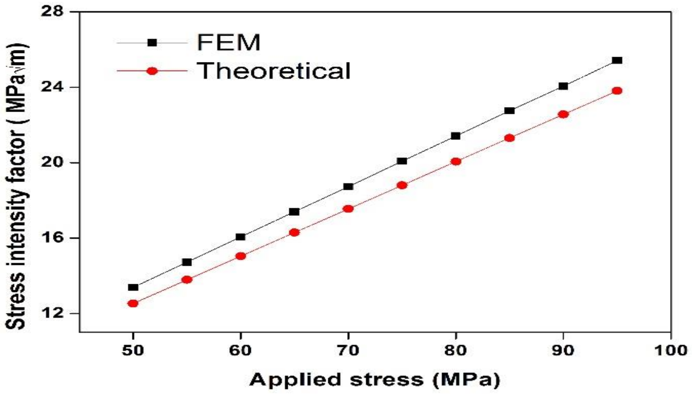
Next we have done simulation on pure copper material for determining its Stress Intensity Factor (SIF) in mode I using simulation tool ANSYS Mechanical APDL. Under the following different variations they are:

a. By varying crack type.
    - Edge crack.
    - Central crack.
    - Circular crack.
b. By varying applied stress.
c. By varying crack length.

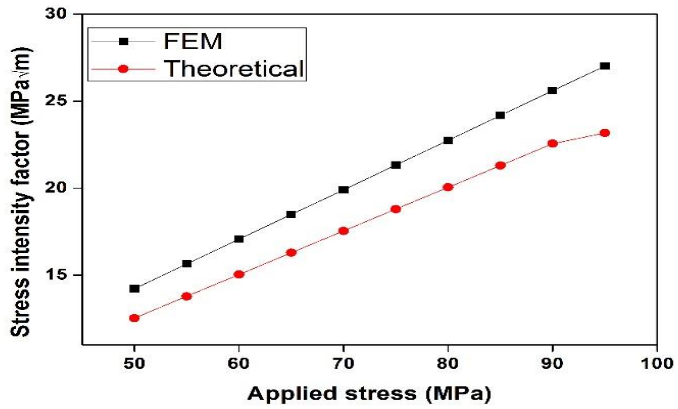
Table 4.3 Values of SIF for varying loading in pure copper material.

S. No	**Applied stress (MPa)**	**$(K_I)_{Theoretical}$ (in $MPa.m^{1/2}$)**	**Edge crack, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Central crack, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Circular crack with an edge, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**
1.	50	12.533	13.384	14.220	15.328
2.	55	13.786	14.723	15.644	16.861
3.	60	15.039	16.061	17.066	18.394
4.	65	16.293	17.400	18.488	19.927
5.	70	17.546	18.730	19.910	21.460
6.	75	18.799	20.076	21.333	22.992
7.	80	20.053	21.415	22.750	24.525
8.	85	21.306	22.750	24.179	26.058
9.	90	22.559	24.060	25.601	27.591
10.	95	23.81	25.420	27.023	29.124

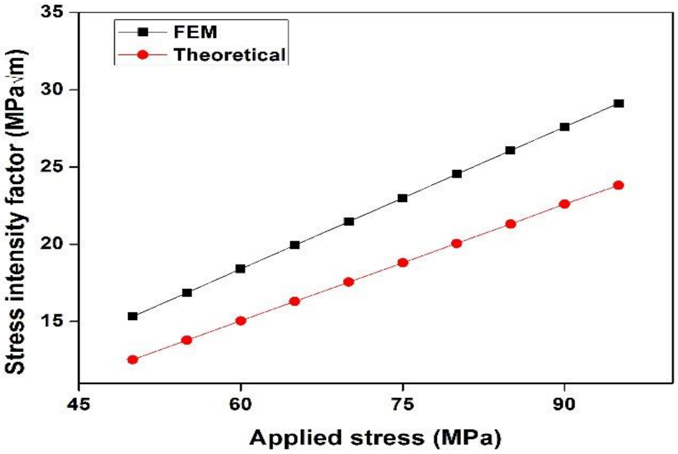
According to Table 4.3, the values of SIF through FEM (Finite Element Method) have found to be increasing with the increase in applied stress. Hence it has proved that K_I *i.e.* Stress Intensity factor around the tip of the crack surface is directly proportional to the applied stress. Again, according to Table 4.3, the values of SIF through FEM have found to be changing with complexity of the crack. Hence it has proved that the geometry of crack influences the value of K_I. Stress Intensity factor has higher values for more complicated geometry because the residual stresses around the crack tip got increased.



(a)



(b)



(c)

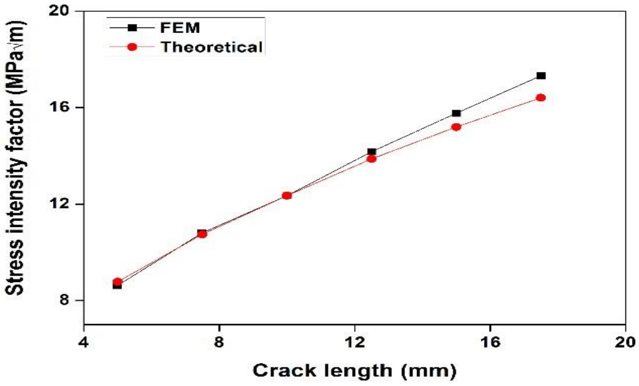
Fig 4.3 Stress Intensity Factor Vs Applied stress graph for pure copper material (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to the Fig 4.3, the curve for theoretical values of K_I has found to be lied above the curve for FEM values of K_I. Also the difference between the corresponding values at the same applied stress is also constant i.e. the curve are almost parallel to each other. Hence it showed that for the copper material the geometrical factor is almost constant in each of the cases.

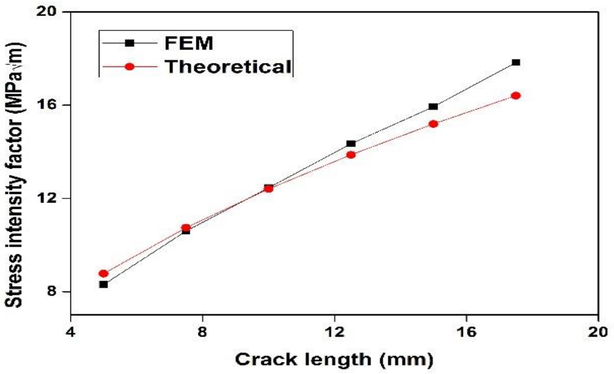
Table 4.4 Values of SIF for varying crack length in pure copper material.

S. No	**Crack length (in mm)**	**$K_{I(Theoretical)}$ (in $MPa.m^{1/2}$)**	**Edge crack $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Central crack $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Circular crack with an edge $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**
1.	5	8.773	8.620	8.368	9.012
2.	7.5	10.744	10.800	10.594	11.185
3.	10	12.407	12.350	12.461	13.280
4.	12.5	13.871	14.167	14.348	15.254
5.	15	15.195	15.770	15.937	17.135
6.	17.5	16.413	17.329	17.837	19.183

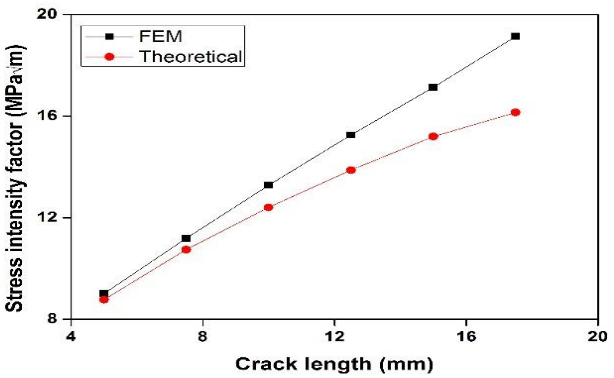
According to the Table 4.4, looking over the values of K_I for a particular kind of crack. It has found to be raising with increment in the crack length of the crack. Hence, for the copper material the dependency of Stress Intensity Factor on the crack length is same as Aluminium i.e. directly proportional. But with the change in the location of crack to change in the value of K_I is in ascending order. It has meant that location changes also affecting to the values of K_I for the copper material.



(a)



(b)



(c)

Fig 4.4 Stress Intensity Factor Vs Crack length graph for pure copper material (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to Fig 4.4, again the curve for theoretical values has found to be lied above the curve for FEM values. Also, variation among the respective values on both the curve is uneven in the case (a) quite close, (b) curves got intersected (c) moving apart with magnitude. From the above interpretation, we could not predict the exact nature efficiently.

- ***Authentication:*** As per previous researches done on both Aluminium and Copper materials respectively the values of mode I Stress Intensity Factor K_I come out to be approx. 0.8 MPa.m$^{1/2}$ [11] for Aluminium and approx. 21 MPa.m$^{1/2}$ [12] for copper. The definition of Stress Intensity Factor has also been satisfied by the trends observed in value of the K_I with all the variations proposed to it. At the last it has been found that the proposed ANSYS codes for the calculation of SIF through FEM could be authenticated for Al-Cu bimetallic material.

4.3 Al-Cu bimetallic

In the end we have done simulation on Al-Cu bimetallic material for determining its Stress Intensity Factor (SIF) in mode I using simulation tool ANSYS Mechanical APDL. Under following different variations they are,

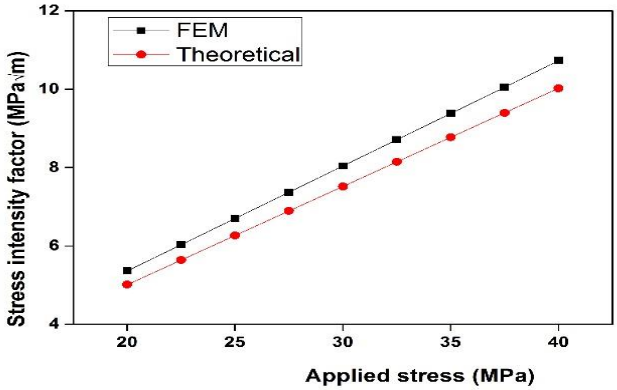
a. By varying crack type.
    - Edge crack.
    - Central crack.
    - Circular crack.
b. By varying applied stress.
c. By varying crack length.

Table 4.5 Values of SIF for varying loading in Al-Cu bimetallic material.

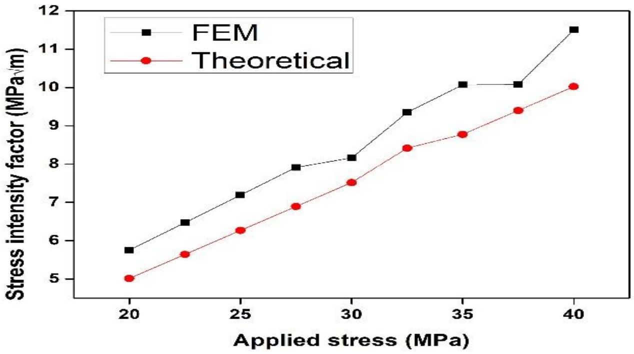
S. No	**Applied stress (MPa)**	**$(K_I)_{Theoretical}$ (in $MPa.m^{1/2}$)**	**Edge crack, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Central crack, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Circular crack with an edge, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**
1.	20	5.013	5.361	5.755	6.139
2.	22.5	5.639	6.032	6.745	6.907
3.	25	6.266	6.702	7.194	7.674
4.	27.5	6.893	7.372	7.913	8.442
5.	30	7.519	8.042	8.633	9.209
6.	32.5	8.146	8.713	9.352	9.976
7.	35	8.773	9.383	10.072	10.744
8.	37.5	9.399	10.052	10.791	11.511
9.	40	10.026	10.723	11.511	12.279

In the Table 4.5, the crack has introduced in Al region of the sheet model. Accordingly values of SIF has calculated. According to Table 4.5, the values of SIF through FEM (Finite Element Method) have found to be increasing with the increase in applied stress. Hence it has proved that K_I *i.e.* Stress Intensity factor around the tip of the crack surface in Al-Cu Bimetal is also directly proportional to the applied stress.

Again, according to Table 4.1, the values of SIF through FEM have found to be changing with complexity of the crack. Hence it has proved that the geometry of crack influences the value of K_I. Stress Intensity factor has higher values for more complicated geometry because the residual stresses around the crack tip got increased.



(a)



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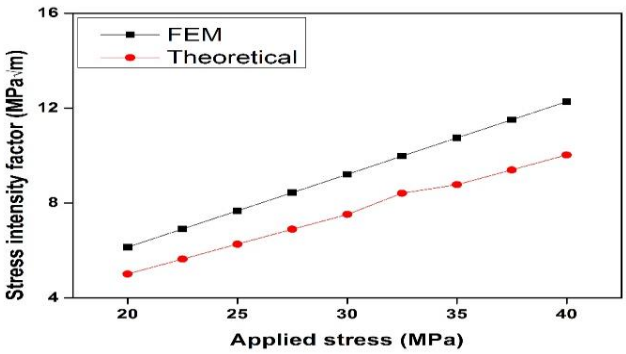


(c)

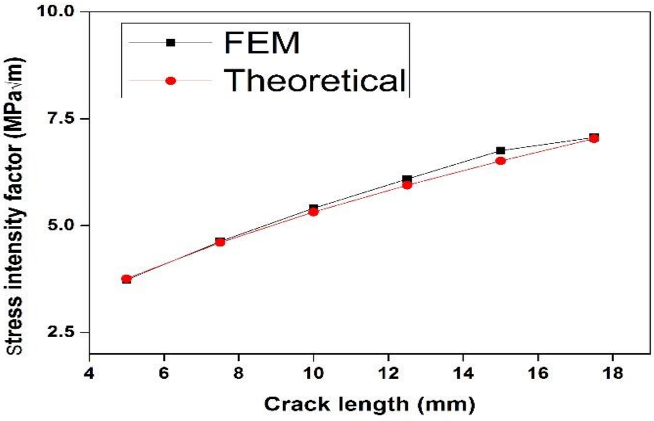
Fig 4.5 Stress Intensity Factor Vs Applied stress graph for Al-Cu bimetallic material (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to Fig.4.5, the curve for theoretical values is below the curve for corresponding FEM values of SIF in each of the case. It can interpreted that not only applied stress but also geometrical factor is also responsible for difference in the SIF values of Al-Cu Bimetals. Location has also affected the difference in the two curves.

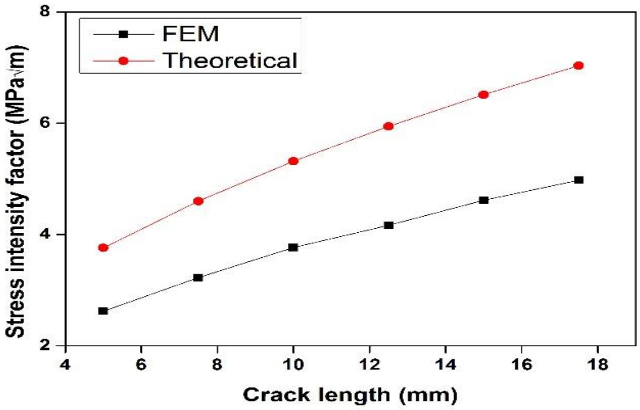
Table 4.6 Values of SIF for varying crack length in Al-Cu bimetallic material.

S. No	Crack length (in mm)	$K_{I(Theoretical)}$ (in $MPa.m^{1/2}$)	Edge crack $K_{I(FEM)}$ (in $MPa.m^{1/2}$)	Central crack $K_{I(FEM)}$ (in $MPa.m^{1/2}$)	Circular crack with an edge $K_{I(FEM)}$ (in $MPa.m^{1/2}$)
1.	5	3.759	3.732	2.623	2.743
2.	7.5	4.604	4.631	3.224	3.352
3	10	5.317	5.406	3.766	3.903
4.	12.5	5.944	6.091	4.167	4.411
5.	15	6.512	6.753	4.614	4.869
6.	17.5	7.034	7.062	4.977	5.315

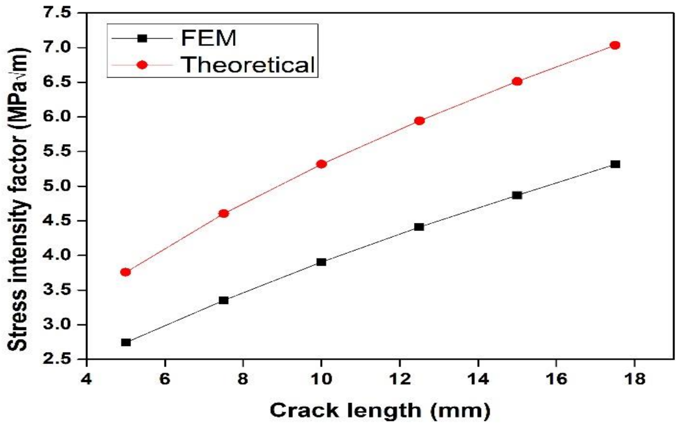
According to the Table 4.6, looking over the values of K_I for a particular kind of crack. It has found to be raising with the increment in the crack length of the crack. Hence, for the Al-Cu Bimetals the dependency of Stress Intensity Factor on the crack length is similar to both the Aluminium or Copper materials. But with the change in the location of crack to change in the value of K_I is in descending order. With increase in the complexity of the crack the values of SIF has found to be decreasing.



(a)



(b)



(c)

Fig 4.6 Stress Intensity Factor Vs Crack length graph for Al-Cu Bimetallic material (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to the Fig. 4.6, the curve for theoretical values found to be above the curve for FEM values of SIF in each of the three cases. It can be said that the geometrical factor has an inverse effect on the value of Stress Intensity factor. So the factor must be in fractional form for Al-Cu Bimetals. Also, variation among the respective values on both the curve is uneven in the following cases (a) quite close, (b) moving apart gradually (c) moving apart gradually with the magnitude.

4.4 Special Case in Al-Cu Bimetallic

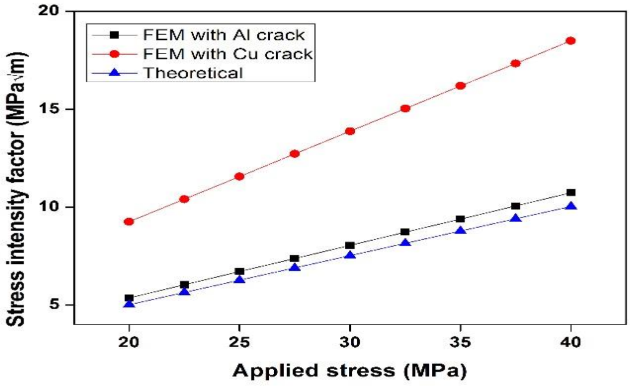
Here we have considered that the crack has been formed in Cu region of the Al-Cu Bimetallic and simulation has been done for determining the Stress Intensity Factor (SIF) using simulation tool ANSYS Mechanical APDL. Under following different variations they are,

a. By varying crack type
    - Edge crack.
    - Central crack
    - Circular crack
b. By varying applied stress
c. By varying crack length

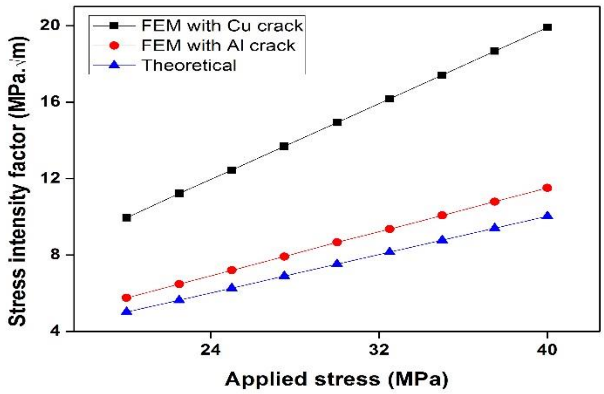
Table 4.7 Values of SIF for varying loading in Al-Cu bimetallic material with crack in Cu region.

S. No	**Applied stress (MPa)**	**$(K_I)_{Theoretical}$ (in $MPa.m^{1/2}$)**	**Edge crack, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Central crack, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**	**Circular crack with an edge, $K_{I(FEM)}$ (in $MPa.m^{1/2}$)**
1.	20	5.013	9.247	9.952	10.310
2.	22.5	5.639	10.403	11.217	11.598
3.	25	6.266	11.559	12.440	12.887
4.	27.5	6.893	12.715	13.684	14.176
5.	30	7.519	13.871	14.928	15.465
6.	32.5	8.146	15.027	16.172	16.754
7.	35	8.773	16.182	17.416	18.042
8.	37.5	9.399	17.330	18.660	19.331
9.	40	10.026	18.494	19.904	20.62

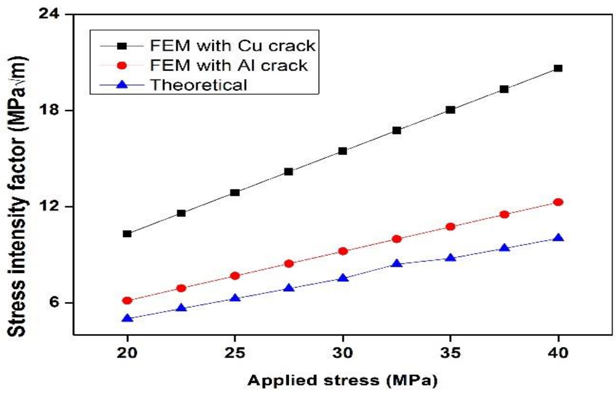
In the Table 4.7, the crack has been introduced in the copper region of the Material. According to the Table 4.7, it is once again clear that the change applied stress value is directly proportional to the value of SIF for Al-Cu Bimetals. Location and geometrical factor has similar influencing trend in this case also. But the values of SIF for Al-Cu material in this case has high magnitude comparatively.



(a)



(b)



(c)

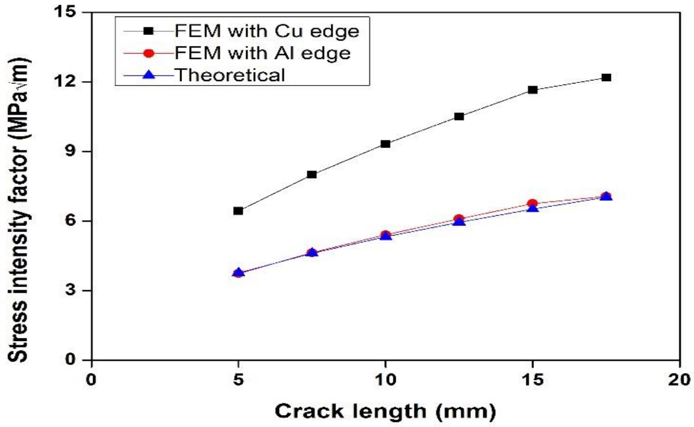
Fig 4.7 Stress Intensity Factor Vs Applied stress graph for Al-Cu bimetallic material with crack in Cu region (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to the Fig 4.7, as expected the curve for theoretical values has found to be below the other two curves for FEM values of SIF of Al-Cu Bimetals. Here in case of Cu region crack curve the difference with other to is very large, therefore the geometrical factor and elastic material properties of the material dependency may also be there.

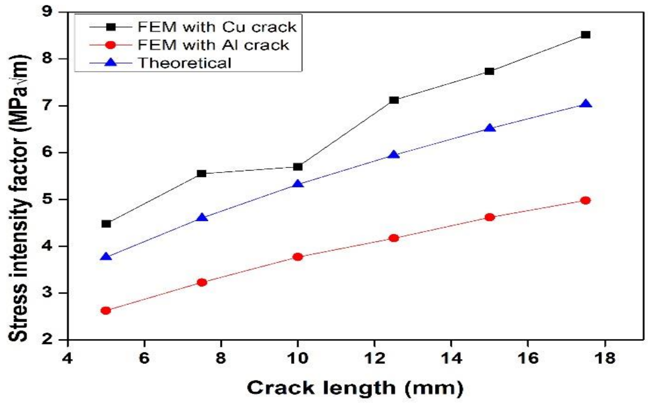
Table 4.8 Values of SIF for varying crack length in Al-Cu bimetallic material with crack in Cu region.

S. No	Crack length (in mm)	$K_{I(Theoretical)}$ (in MPa.m$^{1/2}$)	Edge crack $K_{I(FEM)}$ (in MPa.m$^{1/2}$)	Central crack $K_{I(FEM)}$ (in MPa.m$^{1/2}$)	Circular crack $K_{I(FEM)}$ (in MPa.m$^{1/2}$)
1.	5	3.759	6.437	4.476	4.732
2.	7.5	4.604	7.988	5.547	5.782
3	10	5.317	9.324	5.691	6.732
4.	12.5	5.944	10.505	7.119	7.608
5.	15	6.512	11.646	7.732	8.239
6.	17.5	7.034	12.180	8.511	9.167

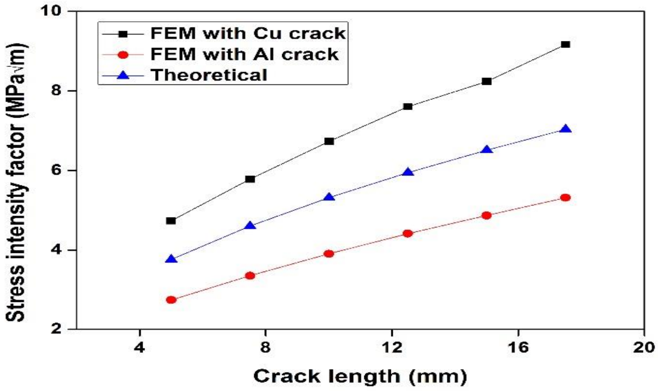
According to the Table 4.8, again looking over the values of K_I for a particular kind of crack. It is found to be raising with the increment in the crack length of the crack. Hence, for the Al-Cu Bimetals the dependency of Stress Intensity Factor on the crack length is similar to both the Aluminium or Copper materials. But with the change in the location and geometry of the crack to the change in the value of K_I is in descending order. With increase in the complexity of the crack the values of SIF has found to be decreasing again.



(a)



(b)



(c)

Fig 4.8 Stress Intensity Factor Vs Crack length graph for Al-Cu bimetallic material with crack in Cu region (a) Edge crack (b) Central crack (c) Circular crack with an edge.

According to Fig. 4.8, the curve for theoretical values have found to be above the curve with Al crack and below the curve with Cu crack for FEM values of Stress Intensity factor. It can be said that the geometrical factor has an inverse effect on the value of Stress Intensity factor with Al crack while direct effect with Cu crack. So the factor must be in fractional form for Al-Cu Bimetals.

In the end, it could be said that the value of Stress Intensity factor for Al-Cu bimetallic material on an average lies around 9 $MPa.m^{1/2}$ with Al crack and 16 $MPa.m^{1/2}$ with Cu crack approximately.

CHAPTER 5

CONCLUSIONS

In the end, from the above obtained trends in the tables and graphs for pure aluminium, pure copper and Al-Cu material we come on to following conclusions.

- On the basis of **applied stress** variations, the values of SIF through FEM (Finite Element Method) is found to be increasing with the increase in applied stress for both aluminium and copper. Hence it concludes that K_I *i.e.* Stress Intensity factor around the tip of the crack surface is directly proportional to the applied stress. Also, the values of SIF through FEM have found to be changing with complexity of the crack. Hence it concludes that the geometry of crack influences the value of K_I. The curve for theoretical values of K_I in each of the case is appear to be lying above the curve for FEM values of K_I. It is due to omission of geometrical factor in the theoretical calculations of Stress Intensity factor K_I.

- On the basis of **crack length** variations, the values of K_I through FEM have found to be increasing gradually with the increase in crack length for both aluminium and copper. Hence it concludes that the value of K_I is directly proportional to the crack length. Therefore, magnitude of Stress intensity factor depends on the crack length. Also as the crack geometry is changed with the each case values of K_I have found to be changed. It mean that the location of crack also plays the role in the Stress Intensity Factor.

Also the curve for theoretical values appears to be lying above the curve for FEM values.

- For **Al-Cu bimetallic material** model, when the crack is in Al region of the sheet model, the values of SIF through FEM (Finite Element Method) have found to be increasing with the increase in applied stress. It means that K_I *i.e.* Stress Intensity factor around the tip of the crack surface in Al-Cu Bimetal is also directly proportional to the applied stress. Also the values of SIF through FEM have found to be changing with complexity of the crack. It means that the geometry of crack influences the value of K_I. Stress Intensity factor has higher values for more complicated geometry.

- But when the crack is in Cu region, abrupt hike in each of the corresponding SIF values is observed. It means in case of Al-Cu bimetallic we have loyalty of having large range of SIF values which will be vital for its applications

- In case of crack length variations in Al-Cu bimetallic material, FEM values for Al region crack is coming less than the theoretical. It means that geometrical factor in this case may be less than 1.

- Finally the SIF value for Al-Cu bimetallic material through ANSYS codes calculations is coming on an average 5 $MPa.m^{1/2}$ for Al region crack and on an average 16 $MPa.m^{1/2}$.

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